Random and Quasi-Random Designs for Quantisation in High Dimension

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Abstract The greedy-packing algorithm for selecting points from a finite (yet large) candidate set is an efficient approach for the incremental construction of space-filling designs. Its computational complexity scales linearly with the design size, and when the algorithm terminates at step k, the resulting design of size k achieves at least 50% efficiency in terms of both packing radius (PR) and covering radius (CR). Furthermore, as the design size grows, these constructions become asymptotically optimal among all incremental methods with respect to the mesh ratio (CR/PR) [2]. When the space $\mathscr{X}$ is a d-dimensional hypercube, variants of the basic algorithm can avoid selecting vertices as design points and prevent excessive clustering near the boundary of $\mathscr{X}$. In high-dimensional settings ($n \approx 2^d$) or extremely high-dimensional ones ($n \ll 2^d$), evaluating space-filling properties via the covering radius loses relevance. Instead, the L_s-mean quantisation error provides a more meaningful metric. Unfortunately, while the greedy minimisation of the L_s-mean quantisation error is computationally feasible in low dimensions [1], it becomes intractable for large d.

In this talk, we focus on random designs composed of i.i.d. points drawn from a suitable distribution. We show that, unless n is astronomically large (so that Zador's theorem [4] applies), random quantisers uniformly distributed on a sphere of appropriate radius achieve exceptional performance for the quantisation of a spherically symmetric distribution [3]. For a d-dimensional hypercube, the uniform distribution can be approximated by a spherically symmetric distribution, and sampling points uniformly (or quasi-uniformly) from a smaller hypercube, or selecting suitable subsets of vertices from an even smaller hypercube, then yields designs with low quantisation error. In this context, greedy packing among a fractional factorial design offers a particularly efficient method for vertex selection.

—Joint work with Anatoly Zhigljavsky, Cardiff University, UK—

References

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